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Volume II

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Edited by
M. Kourkoulos, G. Troulis, C. Tzanakis

ACTES

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DIDACTIQUE DES MATHÉMATIQUES

Volume II

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UNIVERSITÉ DE CRÈTE

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CAMPUS UNIVERSITAIRE DE
RÉTHYMNON, CRÈTE

Sous la direction de
M. Kourkoulos, G. Troulis, C. Tzanakis

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PREFACE

This is the second volume of the Proceedings of the *4th Colloquium on the Didactics of Mathematics* that took place in 22 & 23 April 2005 at the University of Crete in Rethymnon, Crete, Greece. It includes the texts of the presentations given in English, or French during the Colloquium, as well as abstracts in English, or French of the presentations given in Greek. The first volume consists of the original Greek texts, the Greek translation of the invited talks and abstracts in Greek of all presentations given either in English, or French.

The *Colloquia on the Didactics of Mathematics* are organized by the Department of Education of the University of Crete every two years since 1998. Invited talks by distinguished scholars in the field of Mathematics Education are given during these Colloquia and researchers in mathematics education, mathematicians, or teachers of mathematics both from Greece and abroad, give presentations that have been accepted after they have been blindly reviewed by members of the *Scientific Committee*. In the *4th Colloquium*, there were four invited talks given in plenary sessions. Furthermore, it was the first time that the call for papers was announced internationally and there was a wide response from all over the world. Thirty-one out of the fifty-seven submitted papers came from eight European and seven non-European countries, beyond Greece. The papers were subjected to a blind peer review by at least two members of the *International Scientific Committee*, thirty-seven have been finally accepted, and thirty-three of them are included in these Proceedings.

About 300 participants, mathematics teachers at all levels of education, mathematicians, students of mathematics and education, and researchers in mathematics and mathematics education attend these Colloquia. The positive international response to the call for papers and the extensive participation to the *4th Colloquium* have convinced us that organizing these Colloquia is valuable for both the Greek educational community and the research community in the field of Mathematics Education. Therefore, we believe and hope that it can become an established event, organized regularly on a two-year basis.

These *Colloquia* have a three-fold aim: (a) to present current international trends on important issues of the Didactics of Mathematics, with reference to all educational levels, thus giving an opportunity to the educational community to be informed about those issues that are of great importance nowadays; (b) to provide an opportunity for fruitful communication between the Greek researchers of the Didactics of Mathematics and their distinguished colleagues from abroad; (c) to make clear that the problems and basic issues on the teaching and learning of Mathematics at any level of education are interrelated with those at all other levels; and therefore, that the contact, collaboration and mutual understanding among teachers of Mathematics and researchers on Didactics of Mathematics is an essential prerequisite for improving Mathematics Education.

We would like to thank the members of the *International Scientific Committee*, who were willing to review the submitted papers thoroughly. We also thank the Rectorate of the University of Crete and the Commercial Chamber of Rethymnon for financial support, Dr. Y. Thomaidis for providing several bibliographical references that were missing or incomplete, Mrs S. Nikolaidou and F. Toutountzaki for the careful translation of the texts and Mrs C. Kalouda and Th. Zarkadi

for designing the cover of these volumes.

Every paper falls within one, or more of the four sections of this volume. However, for practical reasons, each one is cited in the section with the content of which it is related more: (1) Learning and failure in Mathematics, (2) Epistemological and Methodological issues on Mathematics and its Teaching, (3) Alternative forms of teaching and New Technologies in Mathematics Education Mathematics (4) Aspects of the Didactics of Geometry.

As it has been repeatedly stressed, the interdisciplinary character of the DM requires that both the researcher in this field and the teacher of Mathematics have a deep and comprehensive knowledge beyond Mathematics itself, on such diverse fields like the relationship of Mathematics with the other exact sciences, its history and epistemology, cognitive psychology, applied statistics, new technologies and informatics etc. In particular, it gets clearer the necessity of a closer collaboration among the Departments of Education, the Departments of Mathematics and primary and secondary school teachers of Mathematics. We all desire and wish this volume to be valuable in this direction, always aiming at improving Mathematics Education.

Rethymnon, October 2005

M. Kourkoulos
G. Troulis
C. Tzanakis

AVANT PROPOS

Le présent volume constitue la deuxième partie des actes du *4ème Colloque en Didactique des Mathématiques,* qui a eu lieu à l'Université de Crète, à Rethymnon, les 22 et 23 avril 2005. Il comprend, en anglais ou en français, les exposés entiers des intervenants étrangers, ainsi que des résumés des exposés des intervenants grecs, en anglais ou en français. Le premier volume comprend les textes originaux des exposés grecs, les textes intégraux des intervenants étrangers invités à ce colloque, traduits en grec, ainsi que des résumés en grec des exposés des intervenants étrangers.

Les *Colloques en Didactique des Mathématiques* sont organisés, depuis 1998, tous les deux ans, par le Département de l' Éducation de l'Université de Crète, au campus universitaire, à Rethymnon et ils comprennent des travaux soumis des scientifiques grecs et étrangers, qui ont été admis à la suite d'une procédure de relecture de la part des membres du *Comité Scientifique*. Ils comprennent aussi, des exposés de conférenciers invités distingués. Au cours du *4ème Colloque*, il y a eu quatre conférences plénières de scientifiques étrangers invités, tandis que l'appel à l'intérêt des scientifiques était international, pour la première fois et il y a eu un grand intérêt de nombreux pays étrangers. Cinquante-sept exposés ont été remis dont trente et un venaient de quinze pays étrangers, desquels pays étrangers les huit étaient européens. Les articles des travaux soumis ont été examinés par au moins deux membres du Comité Scientifique International; trente-sept d'eux ont été admis dont trente-trois sont compris dans les actes du *Colloque.*

Chaque fois, presque de 300 participants assistent à ces *Colloques* et on y constate une ample participation de la part de la communauté éducative. Cet intérêt, extrêmement positif, même au niveau international pour ce dernier *Colloque,* en ce qui concerne la participation aussi bien que la remise des études, nous emmène à croire que ce *Colloque* couvre une nécessité réelle de la communauté éducative ainsi que de la communauté de recherche, en ce qui concerne le domaine de l'Éducation Mathématique. Par conséquent, ces *Colloques* en tant qu'une activité constamment organisée tous les deux ans, tendent à constituer une vraie institution au domaine de l'*Éducation Mathématique,* avec des objectifs principaux: (a) présenter des exposés sur les évolutions récentes et internationales en ce qui concerne des sujets importants de la Didactique des Mathématiques à tous les niveaux de l'enseignement, et de cette manière-ci, informer la communauté éducative sur les questions de la Didactique des Mathématiques pour lesquelles il y a actuellement un grand intérêt de recherche; (b) donner la possibilité d'une communication constructive parmi les chercheurs grecs de la Didactique des Mathématiques et leurs collègues distingués à l'étranger; (c) révéler clairement que les questions et les problèmes en ce qui concerne l'enseignement et l'apprentissage des Mathématiques à tous les niveaux de l'enseignement sont communs; par conséquent, le contact, la collaboration ainsi que la compréhension parmi les enseignants et les chercheurs en Didactique des Mathématiques constituent une condition essentielle vers l'amélioration de l'Éducation Mathématique actuelle.

Nous tenons à remercier les membres du *Comité Scientifique International* qui, de grand cœur, ont examiné les articles soumis de façon étayée et constructive. Nous remercions également le Rectorat de l'Université de Crète et la Chambre de Commerce de Réthymnon pour leur support financier à notre effort, M. Y. Thomaidis pour avoir procurer des références bibliographiques qui

manquaient ou qui étaient incomplètes, Mmes S. Nikolaïdou et Th. Toutountzaki pour leurs traductions soignées, ainsi que Mmes K. Kalouda et Th. Zarkadi pour la maquette de la couverture.

Bien que tout exposé corresponde à une ou plusieurs des quatre unités thématiques auxquelles ces deux volumes sont divisés, pour des raisons pratiques, il est compris dans l'unité avec laquelle il se rapporte le plus: (1) Apprentissage et échec en Mathématiques, (2) Questions épistémologiques et méthodologiques concernant les Mathématiques et leur enseignement, (3) Formes alternatives d'enseignement et nouvelles technologies dans l'Éducation Mathématique, (4) Questions sur la Didactique de la Géométrie.

Comme on a déjà souligné à plusieurs reprises, le caractère interdisciplinaire de la Didactique des Mathématiques, aussi bien de la part du chercheur que de la part de l'enseignant, exige des connaissances amples et profondes dans beaucoup d'autres domaines au-delà du domaine des Mathématiques en tant que tels, comme par exemple, les autres sciences exactes, leur histoire et leur épistémologie, la psychologie cognitive, la statistique appliquée, les nouvelles technologies etc. Plus particulièrement, on voit se révéler la nécessité d'un rapprochement plus étroit entre les Départements de l'Éducation et ceux des Mathématiques, ainsi qu'entre ces Départements et les enseignants des deux premiers niveaux d'enseignement. Nous espérons que le présent volume aidera dans ce sens, dans le but de l'amélioration de l'Éducation Mathématique.

Rethymnon, octobre 2005
M. Kourkoulos
G. Troulis
C. Tzanakis

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