4th COLLOQUIUM ON THE DIDACTICS OF MATHEMATICS

(with international participation)

DEPARTMENT OF EDUCATION

UNIVERSITY OF CRETE

FRIDAY - SATURDAY 18 & 19 APRIL 2003

UNIVERSITY CAMPUS

RETHYMNON, CRETE

AUDITORIA Δ6 , Δ7, Γ2

THEMES

The teaching and learning of Geometry
The teaching and learning of Statistics and Probability
The interrelationship between Mathematics and Physics: Didactical issues
The didactical value of the history and epistemology of Mathematics
Possible causes of failure to teach and to learn Mathematics
Alternative methods of teaching Mathematics

INVITED SPEAKERS

F. Furinghetti

C. Hoyles & R. Noss

F. Pluvinage

G. Schubring

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4th COLLOQUIUM ON THE DIDACTICS OF MATHEMATICS

(with international participation)

DEPARTMENT OF EDUCATION

UNIVERSITY OF CRETE

FRIDAY - SATURDAY 22 & 23 APRIL 2005

UNIVERSITY CAMPUS

RETHYMNON, CRETE

AUDITORIA Δ6, Δ7, Γ2

PROGRAMME & ABSTRACTS

TITLES OF PRESENTATIONS OF INVITED LECTURES

(**underlined** is the name of the co-author(s) who will present the paper orally and whose name(s) appear(s) on the time schedule of the programme)

Fulvia Furinghetti & **Annamaria Somaglia**: *The history of mathematics and teacher education in practice: a case study*

Celia Hoyles & **Richard Noss**: *Designing Mathematical Learning Environments for Collaboration at a Distance*

François Pluvinage: *Mathématiques d'un point de vue didactique*

Gert Schubring: *Generalizing the concept of multiplication – Epistemological implications of the relation between quantity and number*

TITLES OF PRESENTATIONS IN PARALLEL SESSIONS

(**underlined** is the name of the co-author(s) who will present the paper orally and whose name(s) appear(s) on the time schedule of the programme)

Ch. Androni, E. Dimitrakopoulou, K. Zaharos: *Social and cultural aspects of failure in mathematics in kindergarten* (**in Greek**)

M. Anido, M. Guzmán, R. Katz: *The comprehension of the concept of vector as a didactical engineering object*

M. Barabash: *Didactics of geometry teaching at school based on teachers' systematic knowledge of a relevant geometrical theory*

R. Bkouche: *La Géométrie entre mathématiques et sciences physiques*

M. Bonacina, A. Haidar, M. Quiroga, E. Sorribas, C. Teti, G. Paván: *Mathematical teaching-learning and the development of "sociomathematical" norms*

C. Bonotto: *Mathematizing the everyday or 'everydaying' mathematics?*

E. Demetriadou: *The effectiveness of 15 year-old students in basic geometrical constructions* (**in Greek**)

D. Escobar: *Teaching probability and statistics for different disciplines*

J.L. Galán, M.A. Galán, A. Gálvez, A.J. Jiménez, Y. Padilla, P. Rodríguez: *Programming with CAS as an alternative method of teaching mathematics in engineering*

B. Georgiadou-Kambouridou & **Ch. Bakas**: *Analyzing a teaching experiment in Geometry with 5th grade primary school pupils* (**in Greek**)

I. Georgiou, M. Kaisari, T. Patronis: *The concepts of vector and parallelogram, their transformations and didactics: An experiment with prospective high school teachers* (**in Greek**)

Z. Gooya & B. Z. Zangeneh: *How teachers conceive geometry teaching in Iran*

G. Halepaki: *Abilities and behaviour of 6th grade primary school students with regard to the estimation and checking criteria of the magnitude of the operation results* (**in Greek**)

S. Igliori, L.F. Godoy: *A study of factors associated with difficulties in understanding the derivative concept*

M. Kaldrymidou, M. Tzekaki, Ch. Sakonidis: *The management of the construction of meaning in the mathematics classroom* (**in Greek**)

M. Kourkoulos & **E. Mantadakis**: *Eléments sur le comportement des étudiants des Sciences de l'Education concernant la compréhension de la notion de la dispersion dans le cadre de la statistique descriptive* (**in Greek**)

Ch. Lemonidis, M. Hatziliami: *Family's functional characteristics and the arithmetical knowledge of preschoolers* (**in Greek**)

P. Linardakis: *Didactics of Mathematics and Lexicography* (**in Greek**)

C. Maranhão, L. Martinelli, S. Sentelhas: *Knowledge of ordering relations in high school*

Ch. Mitsoullis: *What a mathematics teacher has learned from his 12 year-old high school pupils when teaching them the concept of angle by using concrete material* (**in Greek**)

N. Mousoulides, M. Pittalis, C. Christou: *Development of an intervention project for teaching problem-solving* (**in Greek**)

K. Nikolantonakis : *La multiplication dans le cadre de la formation continue des professeurs d'écoles* (**in Greek**)

G. Noël: *The use of probability as a source of problems in mathematics teaching*

R. Ovodenko & P. Tsamir: *Possible causes of failure when handling the notion of inflection point*

M. E. Paradise: *Developmental Mathematics*

G. Perikleidakis: *The understanding and the resolution of verbal problems by elementary school pupils with learning disabilities in mathematics: An experimental teaching* (**in Greek**)

G. Polyzois: *Planning a software-assisted instructive intervention in problems of navigation, suitable for children 5-7 years old* (**in Greek**)

C. Sárvári: *Pragmatic, epistemological, and heuristic values in CAS enhanced mathematics education*

E. Saucan: *A place for differential geometry?*

A. Strantzalos: *A proposal for a "change of framework" of the reasoning procedures used in high-school Euclidean Geometry, motivated by Arcimedes' work "On Plane Equilibriums"* (**in Greek**)

P. Strantzalos: *A new approach to the teaching of Euclidean Geometry to students of the 1st class of the Greek Lyceum* (**in Greek**)

D. Tanguay: *Une expérimentation sur l'apprentissage de la structure déductive en démonstration*

E. Theodorou & Ch. Lemonidis: *A new interdisciplinary proposal for teaching Geometry to lower elementary school* (**in Greek**)

Y. Thomaidis & M. Stafylidou: *A research on the perspectives and possibilities of a cross-curricular teaching approach: The case of Euclidean Geometry in the 1st class of the Greek Lyceum* (**in Greek**)

G.M. Troulis: *Interdisciplinarité et Mathématiques: Exemples de modélisation* (**in Greek**)

H. Vasilaki, A. Spyridakis, J. Stamelos, E. Yachnakis, J. Kanellos: *Test anxiety and metacognitive skills* (**in Greek**)

L. Venegas: *Une réponse possible au manque de motivation envers les mathématiques*

ABSTRACTS OF INVITED TALKS

The history of mathematics and teacher education in practice: a case study

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Annamaria Somaglia, Liceo Scientifico 'Convitto C. Colombo', Genova, Italy

In this paper we consider the problem of the existing prospective teachers' beliefs about the teaching of mathematics, in particular, their belief that it is ineluctable to reproduce in their teaching practice the style of mathematics teaching they have seen in their school days. We think that the prospective teachers need of a lens allowing them to look at the topics they will teach in a different manner. This lens may be provided by the history of mathematics. In this paper we shall discuss an example of constructing a teaching sequence in algebra through the history that was carried out in the laboratory of mathematics education within the frame of a course for perspective teachers. The particular conditions of our experiment, notably the fact that our prospective teachers had not a specific preparation on the history of mathematics, allow to outline chances and caveats of the use of history in teacher training.

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Designing Mathematical Learning Environments for Collaboration at a Distance

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Drawing on examples and insights derived from the EU-funded project, WebLabs, we focus on a set of epistemological and didactical issues, which arise in the context of students in different countries designing, building and sharing models of their evolving mathematical knowledge. WebLabs has built sets of working (re)programmable tools through which students have been able to explore mathematical ideas and share them through specially designed WebReports.

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Mathématiques d'un point de vue didactique

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Je résume ci-après la présentation que j'envisage.
1 - Introduction: la différence de vision selon que l'on considère une question de mathématiques pour l'étudier en elle-même ou que l'on envisage de l'inclure dans un enseignement.
2 - Quelques principes généraux a propos de didactique des mathématiques (Freudenthal, Brousseau, Artigue, Duval).
3 - A propos de la place des élèves, un exemple d'évolution d'un projet associant géométrie dans l'espace et fonctions, depuis son étude a priori jusqu'a sa présentation définitive dans le cadre d'une vidéoconférence.
4 - Exemple d'une analyse a priori non encore (véritablement) expérimentée: a choisir entre la transformation de Bronner et la trisectrice de Maclaurin (les deux conduisent, entre autres, a envisager l'étude de polynômes de degré trois).

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Generalizing the concept of multiplication – Epistemological implications of the relation between quantity and number

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The relation between mathematics history and mathematics teaching constitutes a topic of high actual interest in research on mathematics education. In particular, there are problems inherent to the nature of mathematics and which prove to be relevant in the teaching-learning process that one hopes - sometimes in a too immediate way - to better approach by introducing knowledge from the history of mathematics. It is the concept of "epistemological obstacle" which constitutes a major focus for transposing historical knowledge to teaching. This paper analyzes potentialities and weaknesses of the concept: on the one hand, by following the establishment of this concept within didactical theories, and on the other hand by analyzing processes of historical development within mathematics. New results from research on the concept of multiplication reveal hitherto unknown "ruptures" in mathematical development so that immediate transfers from history to the classrooms should be avoided.

ABSTRACTS OF ORAL PRESENTATIONS IN PARALLEL SESSIONS

Social and cultural aspects of failure in mathematics in kindergarten

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The present research has been carried out with preschool students from two different social-cultural groups: Roma and non-Roma. Students were interviewed individually and attempted to answer to specific mathematical tasks. The present research aims to provide answers to the following questions: whether preschoolers have the abilities to deal with the given mathematical tasks, and in addition whether there are educational differences between the two different social-cultural groups.

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The comprehension of the concept of vector as a didactical engineering object

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Both from reported research and from our own experience, we have noticed that the main difficulty of students in understanding the essential concepts and methods of Linear Algebra lies in the lack of their training to understand an axiomatic theory and the failure to perceive the need for such a theory.
Faced with this situation, we have developed and tested a proposal, within the theoretical framework of *Didactical Engineering*, for an introductory course prior to the Linear Algebra course taught to first-year students at an Engineering School. Students construct the "Euclidean vector space of arrow vectors" (free vectors in space) as a rigorous mathematical theory. The idea is to turn the geometric vector into a visual referent that will make it easier to globally understand problems requiring a higher level of abstraction.
Once the vector space has been constructed, component representations are introduced through the definition of bases in the straight line, the plane and space. Operations in $\mathbf{R}$, $\mathbf{R}^2$ and $\mathbf{R}^3$ are performed on the basis of implicit isomorphisms. The ultimate aim is to help the students to perceive vector spaces as powerful, general tools for use in a variety of applications.

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Didactics of geometry teaching at school based on teachers' systematic knowledge of a relevant geometrical theory

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Structured teaching and learning of theoretical geometry is generally believed to be unattainable for the beginners, i.e. for those students who are still at visual levels of geometrical perception. This belief has led to various attempts to teach geometry on another basis, which is almost purely empirical and does not form a consistent subject-matter foundation for future learning of deductive geometry; neither do it helps in advancing the learners towards higher levels of geometrical thinking needed for the transition to deductive geometry. What is proposed here is an outline of a theoretical-geometrical knowledge, which, on the one hand, is adjustable for beginners, and on the other hand, provides the basis for the future development of one's geometrical abilities. Speaking of beginners, we mean the students of math-teachers education programs for primary and secondary school, but keep in mind their future school pupils. Thus, this approach is both a kind of "geometry for beginners", and a theoretical basis for didactics of geometry teaching. The idea is to start teaching from purely geometrical notions, which are on the one hand easily demonstrated on handy models, such as paper folding, and on the other hand, are consistent with theoretical notions and objects of the so-called synthetic Euclidean geometry.

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La Géométrie entre mathématiques et sciences physiques

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Lorsque nous disons que la géométrie élémentaire se situe au carrefour des sciences mathématiques et des sciences physiques, nous signifions d'une part que les objets de la géométrie ont une origine empirique, d'autre part que leur étude relève de la méthode déductive. En cela la géométrie élémentaire peut être considérée comme participant de la

physique des corps solides. La géométrie élémentaire, sous la forme que lui a donnée Euclide, apparaît ainsi comme la première étude rationnelle de phénomènes naturels (les corps solides), devenant ainsi un modèle lorsque, avec la révolution scientifique du XVIIe siècle, la physique est devenue un chapitre des mathématiques, le développement de la physique s'inscrivant dans la continuité de l'œuvre euclidienne d'une part et d'autre part pouvant être considéré comme la réalisation du programme des *Seconds Analytiques*. Une telle conception implique que l'enseignement de la géométrie élémentaire participe à la fois de l'enseignement des sciences mathématiques et de l'enseignement des sciences physiques.

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Mathematical teaching-learning and the development of "sociomathematical" norms

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Avant-garde trends in Mathematical Education comment on the existence of a conflict between "*mathematical work*" and the "*school Mathematics*". They also observe that to solve this, it is necessary to consider *man-performing Mathematics instead of the mathematical production performed by man.*
On the other hand, the nature of the difficulties under scrutiny shows the need to search for the cause of the previously identified problem in other fields.
Thus, we have decided:
∗ to focus our search on the *'intersubjective and sociocultural context in which every individual develops his life, its influence on the genesis of his knowledge structures and the possibility of exercising them'*;
∗ to limit the scope of our investigation to the impact in the teaching of Mathematics caused by the new technologies of information, communication and calculation;
∗ to set the following objectives for this first stage:
» searching and developing criteria, sociomathematical norms to establish a system of interpretation in order to reform practice itself.
» monitoring and interpreting the 'theory-practice' conflict, based on the work with materials designed on the previous stage.
We introduce in this work the results of our investigation, acknowledging the fact that the topic needs further investigation and that it might be the ultimate reason for such a widespread school failure.

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Mathematizing the everyday or 'everydaying' mathematics?

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In this paper we will present preliminary results of a study which is part of an ongoing research project aimed at showing how the use of suitable cultural artifacts can play a fundamental role in bringing students' out-of-school reasoning experiences into play, by creating a new tension between school mathematics and everyday-life knowledge with its incorporated mathematics. In this study we decided to use as cultural artifact a weekly TV guide issued as a supplement of a well-known daily paper, in order i) to extend students' capacity to calculate from base 10 to base 12, 24 or 60, ii) to develop the concept of equivalence between time intervals expressed in different ways (days, hours, minutes), iii) to introduce informally the concept of fractions. Besides the use of suitable cultural artifacts and the application of a variety of complementary, integrated and interactive instructional techniques, the teaching/learning environment designed and implemented in this study is characterized by an attempt to establish a new classroom culture also through new socio-mathematical norms. The focus is on fostering a mindful approach toward realistic mathematical modeling which is both real world-based and quantitatively constrained sense-making.

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The effectiveness of 15 year-old students in basic geometrical constructions

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In this work, we discuss the ability of 15 year-old students to perform basic geometrical constructions, based on a comparative study between teachers' views on this issue and an empirical study with students after the 9-year compulsory part of Greek school education (15 year-old students). Our results indicate that many students, after three years of geometry teaching in high school, are not able to make precise constructions by using appropriate

geometrical instruments. Instead, they use empirical methods (e.g. intuition, approximation, measuring). In some cases, their failure is related to ignorance or misunderstanding of geometrical concepts. The results are mainly due to the fragmented teaching of basic geometrical constructions in the first grades of high school.

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Teaching probability and statistics for different disciplines

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Teaching Statistics for different disciplines has raised the question if the depth, the topics, the intensity and the vision is or should be something different according to each area of study. Taking into account these criteria, we attempt in this paper to organize the different courses given by a Mathematics Department to other Programs. The division does not include only "what to give" but also "how to give" and aspects that has to be recognized from the perspective of students, of the colleagues of other disciplines and from us as mathematicians. There is no universal answer to this problematic because the conditions and environment in different countries and universities is different, but nevertheless, we hope to contribute in throwing some light to what is common in different countries and to find or approach appropriate solutions.

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Programming with CAS as an alternative method of teaching mathematics in engineering

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Nowadays, computers are normally still being used in university teaching as great powerful calculators, but not as tools that help to carry out a substantial change in mathematical teaching. The challenge we must face in the future is to overcome this situation in order to use computers as tools for increasing mathematical creativity.
To approach this situation, we can use a programming language in the lectures of Mathematics. When the students program, they must read, raise strategies, modify programs that have already been developed and finally solve the proposed problems with the programs that they themselves have developed. This makes them the protagonists of their learning. In addition, the use of programming allows finding tasks for students that correspond directly to each of the mental constructions of actions, processes, or objects relevant to a mathematical concept. Moreover, a good idea is to complement the programming with CAS. Thus, we attempt to help the student not just solve problems, but also to create the programs that will help to solve these problems. We aim at making the computer a tool that encourages mathematical creativity, and not only a powerful calculator.

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Analyzing a teaching experiment in Geometry with 5th grade primary school pupils

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In this paper we analyze a teaching intervention on the area of plane figures that took place with 5th grade elementary school pupils (10-11year-old). Teaching was based on an open situation problem. A learning environment was created that gave to the children the opportunity to develop different approaches by using their prior geometrical experiences, or by improvising. In the analysis we attempt to identify and comment on the general and particular characteristics of children's solutions regarding the geometrical content and its development. Moreover, the factors that promoted the generation of children's approaches and further possibilities of the teaching intervention are also examined.

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The concepts of vector and parallelogram, their transformations and didactics: An experiment with prospective high school teachers

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As far as the teaching of a mathematical concept (or theory) is concerned at various educational levels, special

questions arise; for example, what are the results of teaching and/or the spontaneous perceptions of the students-future teachers of Mathematics about the same concept (or theory); moreover, how is this specific knowledge connected to the rest of the students' mathematical knowledge. In attempting to answer these questions, we present a comparative study of two teaching experiments, referring to school knowledge of parallelograms and of what is known as *Thales Theorem,* from the viewpoint of Affine Geometry. These experiments took place at the University of Patras, but at different times and under different forms. The first experiment, in which traditional teaching was applied, included mainly the elements of the Theory of Affine Spaces. In the second experiment, applying the Project Method, we asked the students to reorganize the structure of the subject matter of two chapters of the school geometry textbook, mainly the chapter of parallelograms and that of *Thales Theorem,* by using geometric transformations and some specific elements from the Affine Geometry of the Plane. Although teaching involved students' own thinking and ideas, in the first experiment, the students ended by using the *Principle of Chasles* rather mechanistically. The second experiment revealed that the students have minimal experience of the project method; consequently, they did not succeed to take advantage of all elements offered by this method. Because of this lack of experience, the students asked often for help, either directly or not. Nevertheless, the students were led to the study of different areas of mathematics and were encouraged to interact with each other during the project.

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How teachers conceive geometry teaching in Iran

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Since 1992, the centralized system of secondary education in Iran has undergone considerable change. These changes have brought with themselves, major concerns about mathematics teachers' education. The authors of the new geometry textbooks believed that in-service teacher training could and should play a major role in successful implementation of the new changes. The purpose of this paper is to draw upon a study that was designed to investigate the impact of the teacher training sessions on teachers' views about teaching and learning geometry, during the summer 1997, by using the data that were collected from various sources during those sessions

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Abilities and behaviour of 6th grade primary school students with regard to the estimation and checking criteria of the magnitude of the operation results

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According to didactic research, the construction and use of control and estimation criteria of the four arithmetical operations is a very beneficial activity for students learning. However, their role is ignored to a large extent, both by teachers and by curriculum-designers. Taking into account these facts, we made a didactic intervention on sixth grade students of a Greek primary school (experimental group), in order to investigate their behavior and their ability with regard to the checking and control criteria of the order of magnitude estimate of the result of arithmetic operations. A test was given to the experimental group, while a control group was checked, composed of other students of the same grade. Our results indicate that on the one hand there is a great deficit with regard to the use of the estimation and checking criteria by the students of the control group, and on the other hand there is an important improvement of the students of the experimental group.

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A study of factors associated with difficulties in understanding the derivative concept

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This paper presents results from a diagnostic study on the difficulties generated in the teaching/learning process of the concept of the derivative, which emerge from the students' confusion between the meaning and the semiotic representation of this concept. The difficulties detected in this study and pointed out in this article came out from the answers obtained by conducting a series of tests and interviews with a total number of 168 students, among them 78 second-year Engineering undergraduates and the remaining ones from a teaching instruction course for future Mathematics teachers. All students had already dealt with derivatives. This study is based on the *Representation Registers* theory developed by Raymond Duval. The results were evaluated by qualitative data, which can be summarized as follows:
The figural representation of the derivative is interpreted as a "tangent line"; the natural language representation "derivative", is interpreted as a derivative of a function at a certain point, or as a derivative function, and the

symbolic representation, such as $\frac{dy(x)}{dx}$, $y'(x)$, $f'(x)$ could get different meanings.

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The management of the construction of meaning in the mathematics classroom

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The five studies presented here focus on aspects of teachers' management of the construction of meaning in the mathematics classroom: the handling of the epistemological features of mathematics, the ways of dealing with pupils' work and errors and the communicative patterns adopted by teachers. The results show that the management of the content of the subject matter often distorts the mathematical meaning and it is dialectically related to the communicative practices employed.

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Eléments sur le comportement des étudiants des Sciences de l'Education concernant la compréhension de la notion de la dispersion dans le cadre de la statistique descriptive

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Dans cet article nous examinons les difficultés rencontrées par des étudiants des Sciences de l'Education concernant la compréhension de la notion de la dispersion et de ses mesures, dans le cadre d'un cours de la statistique descriptive. L'analyse du comportement des étudiants montre que les plus importantes de ces difficultés sont des difficultés conceptuelles et qu'il n'est pas facile pour l'enseignement d'aider efficacement les étudiants à les dépasser. Ceci, malgré le fait que la statistique descriptive est le cadre le plus simple dans lequel on rencontre la notion de la dispersion. Il faut noter que les difficultés les plus persistantes sont celles qui concernent la *variance*.
L'analyse du comportement des étudiants, combinée avec l'analyse épistémologique du sujet conduit à la thèse que, l'emploi des modèles interprétatifs adéquats, en particulier provenant de la Physique, pourraient aider les étudiants à faire face à ces difficultés et à comprendre plus profondément la notion de la dispersion. A la suite de l'article, nous examinons certains de ces modèles, et en particulier ceux dans lesquels la *variance* peut être interpréter comme énergie moyen (cinétique ou potentielle). En parallèle on analyse les possibilités didactiques qu'offre l'emploi des tels modèles dans le cadre d'un cours introductif de la statistique.

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Family's functional characteristics and the arithmetical knowledge of preschoolers

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This paper presents the results of a research about the relation between some functional characteristics of the family and the arithmetical knowledge of preschoolers. To examine these characteristics of the family, we focus on 3 factors: The stimuli that the environment of a child offers, the attitude and parents' participation concerning these stimuli, and parents' attitude towards learning in general, and the knowledge of their children, in particular. To examine the arithmetical competency of children we chose two topics: oral counting and reading.
The results of this study show that there is a positive correlation between the functional characteristics of the family and the arithmetical knowledge of preschoolers. This means that children who are able to count orally and can recognize many numbers come from families that participate actively in their children's everyday life.

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Didactics of Mathematics and Lexicography

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The aim of this lecture is to present the preoccupations and the choices that were made while writing the mathematical entries in the «Dictionary for the School and the Office» by professor G. Babiniotis.
First we analyze how the included words were chosen, and the difficulties that came up due to the differentiation that exists between the theory of the Lexicography regarding the general lexicons and the approaches of a researcher of the Didactics of Mathematics.
Next, within the framework of the Didactics of Mathematics, we analyze the reasons for which, in some cases, the

definitions were chosen to be more "rigorous" in terms of common sense, whereas in some others, they were chosen to be less "rigorous" and more direct. In both cases, the aim is to set some limits to the "conventions" that are created by school mathematics, as well as, to the way that some mathematical terms are used in everyday life.
We also analyze terms like "School Mathematics", "Obstacle", and "Didactical Contract", which form the theoretical framework of this paper.

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Knowledge of ordering relations in high school

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This paper introduces a research on 15-17 year-old high school students' knowledge of *relations and ordering relations in time*. The study consisted of three stages. In the first one, 3 groups of students (34 first graders, 26 second graders, and 26 third graders of a high school in the state of São Paulo) solved problems involving relations and ordering relations during a test. Results showed that students used the relation "arriving before" as having only the meaning of "arriving immediately before" (restricted meaning) and the ordering relation "not arriving after" (restricted meaning) as "arriving before, or at the same time as" (mathematical meaning). At the second stage, first-graders were submitted to a didactical intervention aiming at overcoming their restricted conceptions of time ordering. At the third one, these first-graders solved problems (similar to the ones used at the first stage) in a post-test activity. We analyze the effect of the didactical intervention by comparing students' performance in the first and third stages (pre and post-tests). Results show that students' knowledge was improved, because they showed a broader, mathematical understanding of these relations.

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What a mathematics teacher has learned from his 12 years old high school pupils when teaching them the concept of angle by using concrete material

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This presentation describes the process of planning and developing, as well as the preliminary results, of a pilot research. The aim of this research is to explore the conditions, the limitations and the effectiveness of the use of concrete materials (structured or unstructured materials, manipulatives, natural and other objects and materials) in the process of teaching-learning the concept of angle in an ordinary first grade class of a middle level public school. In accordance with results mentioned in the international bibliography, preliminary findings have shown that the use of concrete materials has advantages and disadvantages. Therefore, it is necessary for the teacher before introducing them in the classroom, to know and take into consideration the limitations that accompany these concrete materials, which are related to their representational characteristics (nature, form), their representational function (epistemic fidelity, transparency) and their mediated role (mediated activity).

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Development of an intervention project for teaching problem-solving

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This paper presents the design and the results of an intervention project that aimed at providing a framework for teaching problem solving. The development of the project was based on the principles of the RME (Realistic Mathematics Education problem solving project), which used real-life situations for teaching mathematical concepts and the problem solving abilities as defined in the experimental structuralism theory (SCS systems). Fundamental aspect of this project was the integration of contemporary technological tools (e.g. Dynamic Geometry Software and spreadsheets). Two groups of 6^{th} grade students participated in the project: An experimental (135 students) and a control group (141 students). The results showed that the experimental group of students outperformed their counterparts in the control group, both in the problem solving ability and in two of the five SCS systems of cognitive development (Qualitative-Analytic and Causal-Experimental), than the control group.

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La multiplication dans le cadre de la formation continue des Professeurs d'écoles

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L'affirmation que les mathématiques ont une histoire modifie considérablement l'image de cette science auprès des enseignants : les mathématiques ne se résument plus à un produit fini, mais sont un processus dans le temps et dans l'espace. Donner une image constructiviste des mathématiques et permettre une approche culturelle de cette discipline, c'est le but de l'affirmation précédente et aussi l'objectif de notre intervention.
Diverses techniques opératoires pour effectuer la multiplication sont proposées dans les pays différents et dans les époques historiques différentes: la technique arabe, la technique grecque, la technique chinoise, la technique à la russe et égyptienne, la multiplication per gelosia et per crocetta entre autres.
La petite Arithmo-histoire au début de notre intervention dans le cadre de la formation continue des Professeurs d'écoles (basée sur des éléments vrais) et les activités supplémentaires, permet de consolider et de prolonger leurs acquis concernant les techniques opératoires sur la multiplication des nombres entiers. Au travers des différentes techniques opératoires les Professeurs d'écoles s'approprient le sens de l'algorithme de la multiplication. Ils utilisent les différentes techniques (Egyptienne-Russe et per crocetta dans notre texte) pour effectuer la multiplication. Grâce aux activités, ils découvrent la non unicité des procédures pour trouver une solution et ils apprennent à comparer, à confronter, à argumenter et à penser les mathématiques par une perspective historique.

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The use of probability as a source of problems in mathematics teaching

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We consider the problem of the player's ruin and show how to solve it both by an experimental method and by theoretical processes. This approach can be transferred to other similar situations and provoke such activities as modelization, simulation, comparison. . . The level of mathematics encountered makes it possible to present that activity to pupils of about 15 year old.

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Possible causes of failure when handling the notion of inflection point

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This paper describes a study that examined the solutions of prospective mathematics teachers when working with the notion of inflection points -- their errors and possible reasons for these errors. In this paper we focus on another aspect of inflection points and we address prospective teachers' difficulties when working with different types of tasks that involve this notion. We use Tall and Vinner's (1981) notions of concept image and concept definition to analyze prospective teachers' correct and incorrect conceptions of inflection points. We examine the question: What concept images and what concept definitions of inflection points can be identified in prospective secondary school mathematics teachers' solutions to verbally and graphically presented tasks? What are possible origins of these images and definitions?
We investigated 56 prospective mathematics teachers, who participated in the course "Didactical Issues of High School Mathematics" (DIM), as part of their studies in a teacher education program for secondary school at Tel Aviv University. All participants had a first degree in mathematics, mathematics education, or computer science.
During the course, the first 15-20 minutes of each 90-minute session were dedicated to the prospective teachers' individual work on worksheets that included mathematical tasks and occasionally also didactical problems. When the prospective teachers submitted their completed worksheets, they usually continued working on certain tasks in small groups, and then all engaged in a concluding, whole-class discussion. Here we examine the prospective teachers' solution to the worksheets that were distributed in the first two lessons and we focus on the participants' reactions to three representations of tasks that addressed the notion of inflection point.
We identified four types of errors in the prospective teachers' solutions to the tasks: Prospective teachers tended to regard (1) $f'(x) = 0$ as a necessary condition for an inflection point, (2) $f''(x) = 0$ as a necessary **and** sufficient condition for an inflection point, (3) the condition that "the graph increases (decreases) before the point and after it" as a sufficient condition for an inflection point. They also pointed to (4) the location "where the graph bends" as an inflection point.
An initial evaluation of the reasons that may underlie the prospective teachers' erroneous solutions yielded that the solutions were mainly related to the prospective teachers' previous knowledge about investigations of functions (errors of Types 1 and 2 as listed above), some errors were rooted in prospective teachers' images associated with

driving or flying (errors of Type 4), while the last type of errors (3) seemed to relate to both the prospective teachers' previous mathematical knowledge and their daily life.

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Developmental Mathematics

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Developmental mathematics was introduced in the American postsecondary institutions and it has taken firm roots on their campuses. The reason for this, professed by the educators who support it, is that a relatively large number of entering students lack the knowledge and skills required to successfully complete even the most elementary course of collegiate-level mathematics. This development has raised several questions. Some university mathematics departments and conservative taxpayers and politicians question, albeit for different reasons, the appropriateness of the presence of such programs on the collegiate level. On the other hand, a good number of educators on all levels, as well as liberal and progressive citizens and politicians support this development. Still, hardly any reliable research indicates its effectiveness. This paper introduces the reasons and manner of the emergence of developmental education in general and developmental mathematics in particular. It includes present practices, contents, organization and delivery, as well as the writer's findings by experience, through anecdotal comments of his colleagues around the country, and by researching the relevant literature.

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The understanding and the resolution of verbal problems by elementary school pupils with learning disabilities in mathematics: An experimental teaching

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An important percentage of pupils with learning disabilities in Mathematics meet severe difficulties in the solution of verbal problems, even at the upper grades of primary school. Here, we present research results concerning an experimental method of teaching which aimed to help pupils overcome difficulties in understanding and solving addition and multiplication problems. The results of this alternative teaching method are evaluated by comparing the performance of two groups of pupils with learning disabilities in mathematics when solving verbal problems of additive and multiplicative structures with small numbers. Two groups of pupils were formed with equalization in pairs by considering three basic factors: a. performance, b. sex and c. general intelligence. Performance of the group of pupils with learning disabilities, who have been taught by this alternative teaching method, are compared before and after teaching. They are also compared to the performances of the group of pupils with no such additional teaching. This experimental teaching method led to the better performance in understanding and solving verbal problems of the group of pupils who were taught.

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Planning a software-assisted instructive intervention in problems of navigation, suitable for children 5-7 years old

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The field of pre-and-prime school education is considered now mature enough for the quest of ways to incorporate in the curriculum activities done with the use of a computer. The educational community of mathematics is confronted, thus, with the need to focus on ways to make computer based environments strengthen children's construction of mathematic conceptions via their interaction with other children and their schoolteachers. In this study of ours we will refer to the planning of an instructive intervention that we have prepared to take place in a kindergarten of 15 children. We will use educational software, which is suitable for activities of orientation and determination of routes (navigation) developing at the same time a scenario that will aim in the exploitation of emergent mathematic concepts.

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Pragmatic, epistemological, and heuristic values in CAS enhanced mathematics education

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Technology applications in general and Computer Algebra Systems in particular disrupt the existing epistemological balance both in theoretical and in practical mathematical work. Owing to the use of CAS modules numerous mathematical procedures are re-evaluated with regard to pragmatic, epistemological, and heuristic values. In this

paper, I will examine the process of re-evaluation and illustrate it with specific examples from calculus.

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A place for differential geometry?

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Differential Geometry of curves and surfaces is of high importance in such fields as Computer Graphics and Computer Aided Geometric Design and their applications. In practice, few Computer Science or Software Engineering students take a Differential Geometry Course, neither do they relate to it. We believe that this is a consequence of the traditional approach that includes a multitude of formulas, systems of partial differential equations and tensors that do not convey geometric intuition and involve a technical apparatus beyond the one acquired in the Calculus courses. We propose an alternative approach, based upon the discrete equivalents of the differential notions. This approach is intuitive and befits the discrete and piecewise-linear contexts. It also allows the student to familiarize early with modern methods and research problems. We show how to introduce various discrete equivalents of such fundamental concepts as Gauss and mean curvature, lines of curvature and geodesics.

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A proposal for a "change of framework" of the reasoning procedures used in high-school Euclidean Geometry, motivated by Arcimedes' work *On Plane Equilibriums*

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We outline a proposal for didactically reshaping the first part of Archimedes' work "On Plane Equilibriums" in the high school geometry curriculum and its didactical implications, aiming to:
-Pointing out the interdisciplinary character of the Euclidean framework, already elaborated since antiquity.
-The introduction of the Euclidean reasoning, through the "proof" of the "law of equilibriums" on the basis of Archimedes' "axioms" of a purely perceptional character.
-The discussion of an example of "abstraction" from the physical notions of "body weight – centre of gravity" and "lever" to the (abstract) notions of "point-weight" and "linear system", respectively.
-The exploitation of these notions, in connection with appropriate "axioms" so that they become embeddable in the Euclidean frame, in order to enrich the methodological tools of the pupils with an alternative, distinguished from the Euclidean, frame of reasoning, which is proved to be effective in a certain group of geometrical problems.
The content of this paper is part of a broader experimentation under progress on the Didactics of the Euclidean Geometry in high school and is composed of two sections: The first concerns Archimedes' work "On Plane Equilibriums" per se and was taught in 2004, while the second one concerns the didactical implications of the first and will be taught during 2005.

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A new approach to the teaching of Euclidean geometry to students of the 1st class of the Greek Lyceum

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We provide a new approach to the teaching of Euclidean Geometry in high school within the framework of "History and Didactics of Mathematics", based on the following two principles:
-Euclidean Geometry is a «cultural asset», and
-it may set free highly qualitative educational elements.
We shall restrict ourselves in the curriculum for the first year of the whole proposal, leaving the rest for another occasion. The presentation will be brief; it will include:
-Indications of the content of the various chapters concerning a short historical account of the cultural developments related to geometry until about 300 A.D.; an indication of the role of axioms in the development of a theory on the basis of the first part of Archimedes' work "On Plane Equilibriums"; Euclid's framework (without measurement); a discussion of the role of measurement in Euclidean geometry; a method of proving geometrical propositions by applying "physical principles" legalized axiomatically; a discussion of the role of the circle in the Euclidean geometry; and a simple characterization of Euclidean in contrast to spherical geometry.
-The fundamental didactical and epistemological principles that determined the whole proposal and have led to the concrete content of the various chapters.
-An indicative sketch of the first chapter and brief guidelines for the teachers, referring to the first, third and seventh chapters. Collaborators of mine cover the rest of the chapters.

This proposal is the object of experimentation in progress.

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Une expérimentation sur l'apprentissage de la structure déductive en démonstration

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La thèse centrale de Duval, à l'égard des difficultés rencontrées par les élèves avec les démonstrations, est à l'effet que ceux-ci n'en saisissent pas facilement les exigences propres parce qu'ils les appréhendent et traitent comme des *argumentations*.
Poussant plus radicalement les orientations de recherche qu'il propose, j'ai conçu des tâches où l'élève organise les propositions d'une démonstration géométrique dont on lui a présenté les grandes lignes, dans les cases vides d'un schéma sagittal. La séquence de tâches a été expérimentée au printemps 2004 à Montréal dans trois classes de 1re secondaire (12-13 ans). Les premières analyses des données recueillies permettent entre autres de conclure que :
- le raisonnement déductif par enchaînement d'inférences n'est pas spontanément compris des élèves, ni dans sa structure locale, ni dans sa structure globale ;
- le passage de la compréhension de prime abord satisfaisante d'une preuve, des idées en cause, de leur articulation dans les grandes lignes, à la production écrite de cette preuve en un raisonnement logiquement bien contrôlé, constitue pour l'élève un saut fondamental, et est intimement lié à sa maîtrise de la structure déductive ;
- le travail d'organisation mis en oeuvre dans les tâches proposées peut contribuer à améliorer l'intelligence qu'a l'élève des mécanismes qui régissent cette structure.

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Ethnomathematics and Geometry: A new interdisciplinary proposal for teaching Geometry to lower elementary school

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Ethnomathematics describes everyday practices of people from different cultural backgrounds that contain some kind of math. The point of view that mathematics is not just an academic activity and do not take place only at school, is not new in the field of math teaching. Using this idea of mathematics as a part of culture, we propose a project that involves multicultural folk art and contemporary art in the teaching of geometry in the first three grades of primary school. The new interdisciplinary curriculum, which is going to be adopted in Greece, offers an ideal framework for this project. In this paper we describe some principles, objectives and examples of activities.
We claim that using multicultural activities for math teaching not only will help students to understand geometrical concepts better, but also will play a vital role in creating a positive attitude towards math.

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A research on the perspectives and possibilities of a cross-curricular teaching approach: The case of Euclidean Geometry in the 1st class of the Greek Lyceum

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During the last two years a research project was realized in the Experimental School of the University of Macedonia, with the approval of the Greek Pedagogical Institute, on the cross-curricular teaching of Euclidean Geometry, Ancient Greek Language and History in the 1st year of Lyceum (15-16 year old students). The basic aim of this project was to examine the possibility of applying the cross-curricular approaches of school knowledge that have been adopted in the new curricula of the nine-years compulsory education, as well as, their potential extension in the Lyceum.
In the cross-curricular courses there have been used worksheets, containing the ancient Greek text of selected geometric propositions of Euclid's "Elements", as well as certain *scholia* on these propositions written by Proklos. In his *scholia*, Proklos includes a lot of the criticism that some ancient Greek philosophers have raised against Euclid. The worksheets were the result of collaboration between mathematics and Greek language teachers, who participated in the project. During the cross-curricular courses, the students had to analyze the ancient texts from a mathematical, literal and historical point of view, guided by their teachers that taught the corresponding school subjects.
In this paper we present the basic principles of the project and its special components concerning Euclidean Geometry, especially the relation between the cross-curricular teaching approach and the traditional aims of teaching this subject. Additionally, we will present certain basic results that concern the problem of interdisciplinary collaboration in school, as well as the effect of the cross-curricular approach in the attitude of students to the

Euclidean Geometry courses.

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Interdisciplinarité et Mathématiques: Exemples de modélisation

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Les programmes analytiques du système éducatif grec ont été constitués, tout au long du vingtième siècle, dans la logique de l'académisme scientifique qui poursuit l'analyse mono-disciplinaire des contenus qui aboutissent à la formation des individus unidimensionnels. Il est intéressant de constater que dans l' aurore du vingt-unième siècle le Ministère de l'Education Nationale a institutionnalisé un nouveau cadre du programme d'études basé sur la logique de l'interdisciplinarité (DEPPS).
Bien que ce programme se trouve sur le stade expérimental, les critiques évaluatives tant par les élèves que par les enseignants et les parents sont très positifs et on espère que l'approche interdisciplinaire à l'école, va renouveler la pratique pédagogique tant sur le plan des relations interpersonnelles que sur le plan des attitudes et les compétences socio-cognitives et les connaissances générales des élèves.
L'approche interdisciplinaire constitue une approche particulièrement privilégiée pour l'enseignement des mathématiques. Par cette approche on espère que les élèves vont comprendre que les mathématiques sont présentes dans tous les domaines de la vie quotidienne, qu'ils sont liés avec d'autres disciplines et qu'on peut trouver différents entrées pour les comprendre.
Notre exposé constitue un essaie de donner quelques exemples d'approcher les maths par la voie interdisciplinaire à l'école primaire.

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Test anxiety and metacognitive skills

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This paper focuses on the relationship between various types of anxiety and metacognitive skills during the completion of a series of mathematical problems. It's well accepted that anxiety affects performance during the completion of cognitive tasks, especially the difficult ones, due to the use of cognitive resources for their processing and completion. Having this theoretical framework in mind, we examined the metacognitive skills of students with high and low levels of anxiety. We examined 278 students aged 12 – 13 years old from different high schools in Crete. The methodological tools used were the following: a) Test Anxiety Inventory by Spielberger et.al., 1970, b) Cognitive Interference Questionnaire by Sarason κ.α., 1996, c) State – Trait Anxiety by Spielberger et.al, 1970. These tools have been standardized for the Greek population and references are available. d) List of metacognitive skills in order to help students to solve the problems. We gave students 4 mathematical problems of two categories, *time – distance* and *fractions - percentages*. Students could solve the first two problems anyway they knew and for the other two had to follow metacognitive instructions. The Cronbach Alpha of the three psychometric instruments used was quite satisfactory. The statistical analysis showed that the students' performance is affected by the two dimensions of anxiety, emotionality and worry, and the cognitive interference. The results of the research project are interpreted and discussed in relation to other results from the international and Greek bibliography.

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Une réponse possible au manque de motivation envers les mathématiques

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En Colombie les mathématiques de la secondaire jouent un rôle crucial au moment où les lycéens doivent définir ses études. À l'université de Los Andes en particulier, les Sciences Sociales, Humaines, Biologiques et de la Santé, de même que les formations Artistiques tels que l'Architecture et les Arts Plastiques doivent le nombre élevé de leurs étudiants en grande partie à l'aridité de cette discipline. En effet, les étudiants de ces domaines si variés ont souvent choisi leur carrière, animés par le désir de montrer qu'ils peuvent très bien réussir dans une des disciplines pas soumises à la dictature des formules, aux raisonnements abstraits et à la simplicité de la pensée binaire. Hélas ! Ignorant que dans les études considérées comme non exactes on a tendance à croire que *la pensée logique*, *le langage formel* et *les outils mathématiques* doivent faire partie de la formation de n'importe quel professionnel, ils découvriront bientôt qu'il est impossible aujourd'hui d'échapper totalement à ce genre de taxes. Un ou deux cours de mathématiques leur donneront la bienvenue dans la première année universitaire, et les étudiants se demanderont ce qu'ils devraient faire enfin pour se débarrasser d'un pareil karma.

Après des essais dans plusieurs directions nous avons cru que le problème principal à ce propos n'est pas un déficit de bases mais plutôt un manque de motivation de la part des étudiants, contraints à subir des programmes où la rigueur logique n'arrive pas à toucher leur sensibilité. C'est pourquoi nous avons décidé de créer des cours où les mathématiques se voient insérées dans un thème qui lui-même attire l'attention des étudiants éloignés des Sciences Exactes, et à travers duquel ils ont fait appel aux concepts, aux algorithmes et aux procédés propres à la géométrie, à l'algèbre et à l'analyse.

Le nombre d'or (ou *la proportion dorée* comme on l'appelle chez nous) fournit un bon exemple d'un pareil cours, car l'architecture et la biologie, la peinture et la musique, la morphologie et la physique, l'histoire et les jeux y trouvent un endroit légitime, qui sert de contexte aux mathématiques tout comme le développement de celles-ci s'avère nécessaire à la compréhension du sujet de l'ensemble. À titre d'exemple prenons une série de Fibonacci, dont les quotients des termes consécutifs, qui tendent d'ailleurs assez vite vers le nombre d'or, servent à illustrer tantôt les oscillations des proportions du corps d'un bébé le long des années jusqu'à l'âge de sa stabilité, tantôt la disposition des feuilles d'une plante autour de la tige. Le premier cas fait intervenir les lois de la morphologie des êtres vivants et le second celles de la phyllotaxis et les deux permettent d'introduire la notion de la spirale logarithmique suggérée à partir des données.

Dans le présent article nous nous proposons de montrer les diverses composantes de ce cours : depuis l'enchaînement du contenu et le système d'évaluation jusqu'aux projets réalisés par les étudiants de différentes disciplines et des plusieurs niveaux dans leurs carrières.